MTH 291 - Differential Equations  
Northern Virginia Community College  
Extended Learning Institute  
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Section 4.8 page 172:2 (Differential Equations with Boundary Value Problems, 7 ed, Zill & Cullen)  

Solve the given system of differential equations by systematic elimination.

\[
\frac{dx}{dt} = 4x + 7y \\
\frac{dy}{dt} = x - 2y
\]

The solution will be of the form \( x(t), y(t) \)  
First, let’s change to operator notation.  
Note: \( D = \frac{d}{dt} \)  
\[Dx = 4x + 7y\]  
\[Dy = x - 2y\]  
\Rightarrow (D - 4)x - 7y = 0  
\[-x + (D + 2)y = 0\]

To eliminate \( x \), we operate on the second equation by \( (D - 4) \).  
\[(D - 4)x - 7y = 0\]  
\[(D - 4)(-x) + (D - 4)(D + 2)y = (D - 4)0\]

Add the two equations.  
\[-7y + (D - 4)(D + 2)y = 0\]  
\[(D^2 - 4D + 2D - 8 - 7)y = 0\]  
\[(D^2 - 2D - 15)y = 0\]  
The auxiliary equation is  
\[m^2 - 2m - 15 = 0\]  
\[(m + 3)(m - 5) = 0\]  
\[m = -3, 5\]  
\[y(t) = c_1 e^{-3t} + c_2 e^{5t}\]  
Now, we can use substitution.  
\[\frac{dy}{dt} = x - 2y \Rightarrow x = \frac{dy}{dt} + 2y\]
\[ x = \frac{d(c_1 e^{-3t} + c_2 e^{5t})}{dt} + 2(c_1 e^{-3t} + c_2 e^{5t}) \]
\[ x = (-3c_1 e^{-3t} + 5c_2 e^{5t}) + 2(c_1 e^{-3t} + c_2 e^{5t}) \]
\[ x = (-3c_1 + 2c_1) e^{-3t} + (5c_2 + 2c_2) e^{5t} \]
\[ x(t) = -c_1 e^{-3t} + 7c_2 e^{5t} \]
\[ y(t) = c_1 e^{-3t} + c_2 e^{5t} \]

Let's have SN check our solution.

\[ \frac{dx}{dt} = 4x + 7y \]
\[ \frac{dy}{dt} = x - 2y \]

, Exact solution is: \( \{ y(t) = \frac{1}{7} C_5 e^{5t} - C_4 e^{-3t}, x(t) = C_4 e^{-3t} + C_5 e^{5t} \} \)

Let \( C_4 = -c_1 \)

Let \( C_5 = 7c_2 \)

Then, \( c_1 = -C_4 \) and \( c_2 = \frac{C_5}{7} \)

Thus, the solutions are equivalent.