MTH 291 - Differential Equations
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Bronson & Costa, page 48, 6.33

Solve the given differential equation.

\[ y' - 7y = \sin 2x \]

Note: You may wish to change the View from 100% to 150%

This is a first-order linear (in \( y \)) ODE. Thus, we can use an integrating factor to solve it.

Can we use separation of variables here?

\[ \frac{dy}{dx} - 7y = \sin 2x \]

\[ \Rightarrow dy - 7ydx = \sin 2xdx \]

It seems that separation of variables will not work here.

An integrating factor is \( e^{\int -7 dx} = e^{-7x} = f(x) \)

We multiply each term of the DE by \( f(x) \)

\[ e^{-7x} \frac{dy}{dx} - e^{-7x} 7y = e^{-7x} \sin 2x \]

The left side of the DE should be of the form:

\[ \frac{d}{dx} \left( \text{dependent variable} \times f(x) \right) \]

\[ \frac{d}{dx} (ye^{-7x}) = e^{-7x} \sin 2x \]

Check this by differentiating to get the left side of the DE.

Multiply each side of the DE by \( dx \)

\[ \Rightarrow d(ye^{-7x}) = e^{-7x} \sin 2xdx \]

Integrate both sides.
\[ \int d(ye^{-7x}) = \int e^{-7x} \sin 2x \, dx \]

\[ ye^{-7x} = -\frac{7}{53} (\cos 2x) e^{-7x} - \frac{7}{53} e^{-7x} \sin 2x + C \]

For the right-hand integral, we can use integration by parts, but I will let SN do the integration for us now.

\[ y(x) = e^{7x} \left( -\frac{7}{53} (\cos 2x) e^{-7x} - \frac{7}{53} e^{-7x} \sin 2x + C \right) \]

\[ \Rightarrow y(x) = -\frac{7}{53} (\cos 2x) - \frac{7}{53} \sin 2x + Ce^{7x} \]

Let's have SN check our solution.

\[ y' - 7y = \sin 2x, \text{ Exact solution is: } \{ Ce^{7x} - \frac{7}{53} \sin 2x - \frac{7}{53} \cos 2x \} \]

Our answers agree!